NPLQCD

YN AND YY INTERACTIONS FROM LATTICE QCD SIMULATIONS

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NPLQCD Collaboration

HYP-XInternational conference of hypernuclear physics,
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MINISTERIO DE CIENCIA E INNOVACIÓN

HadronPhysics I3

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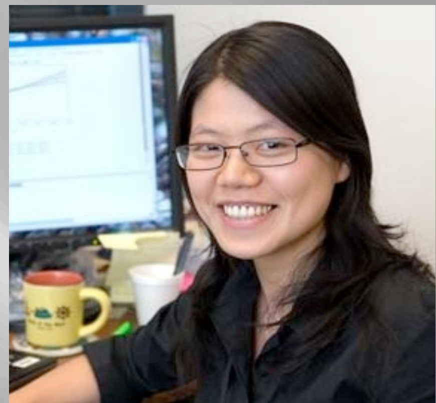

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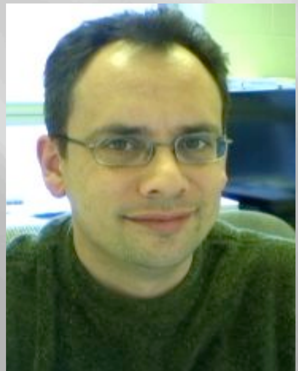

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interaction among hadrons: why lattice?

First principle QCD calculation

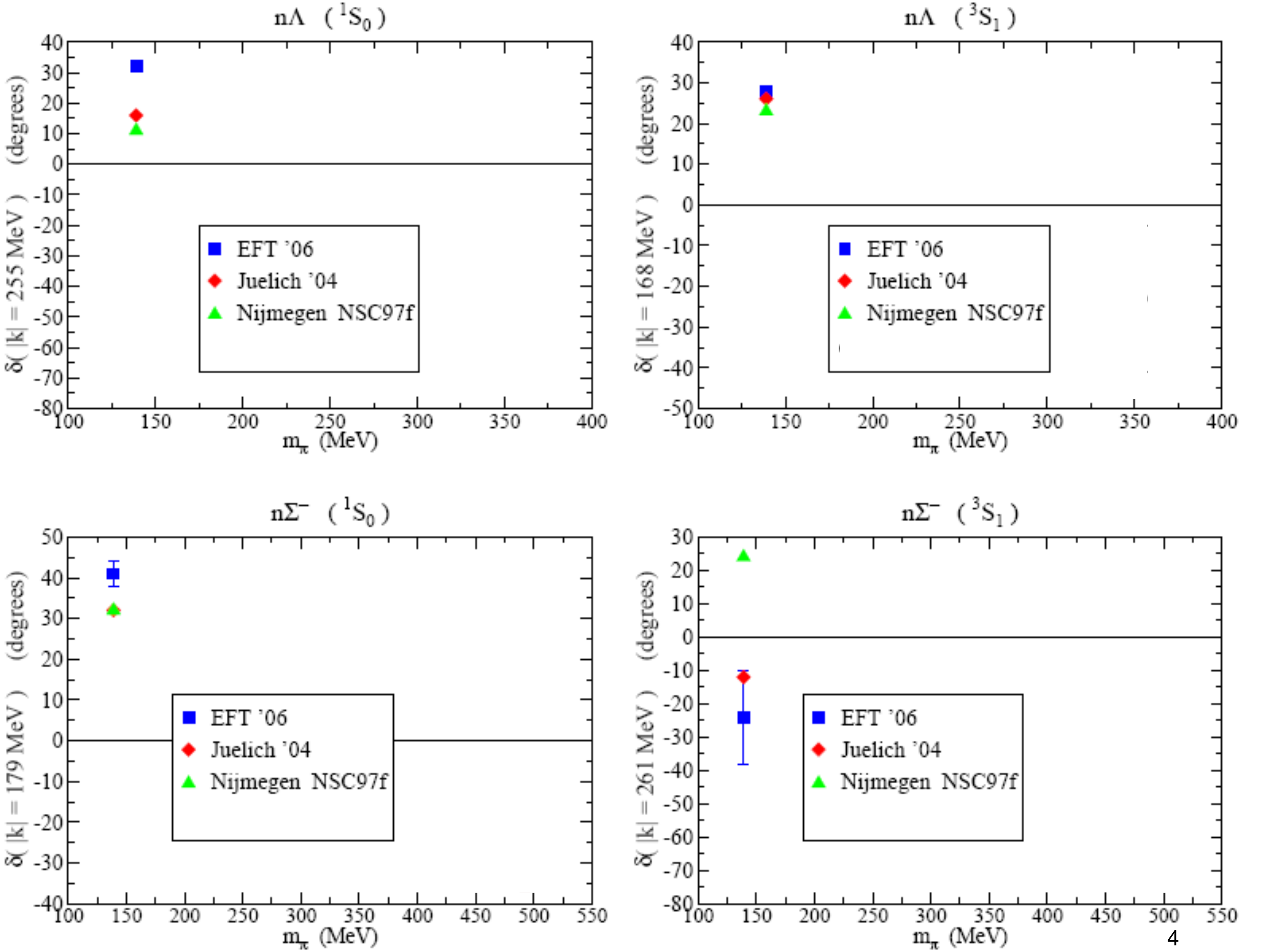
Quantifiable uncertainties

Possibility of study processes which are not accessible experimentally

Examples of the impact of few body lattice simulations:

- Evolution of a supernova (NEOS)
- Nuclear structure calculations
- Hadronic parity-violation

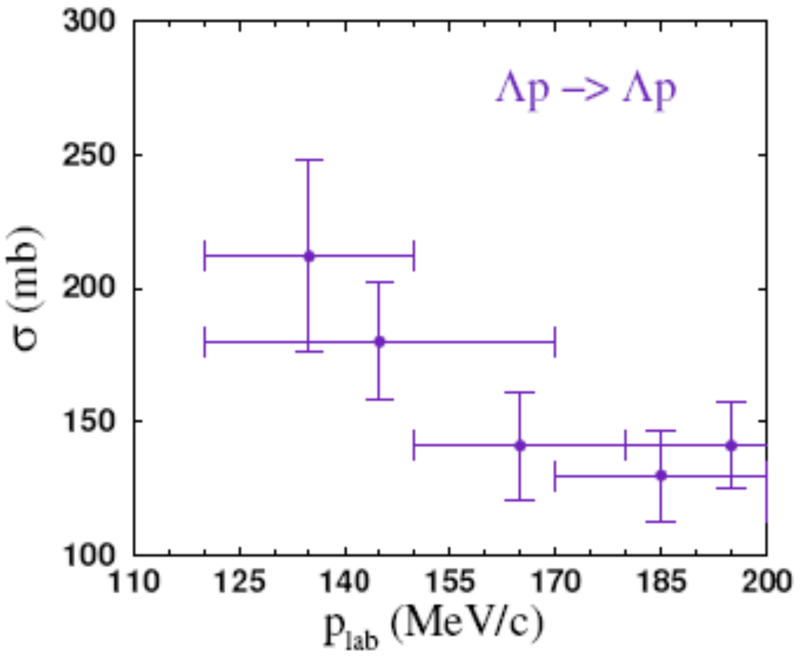
Hypernuclear physics
(structure and decay)

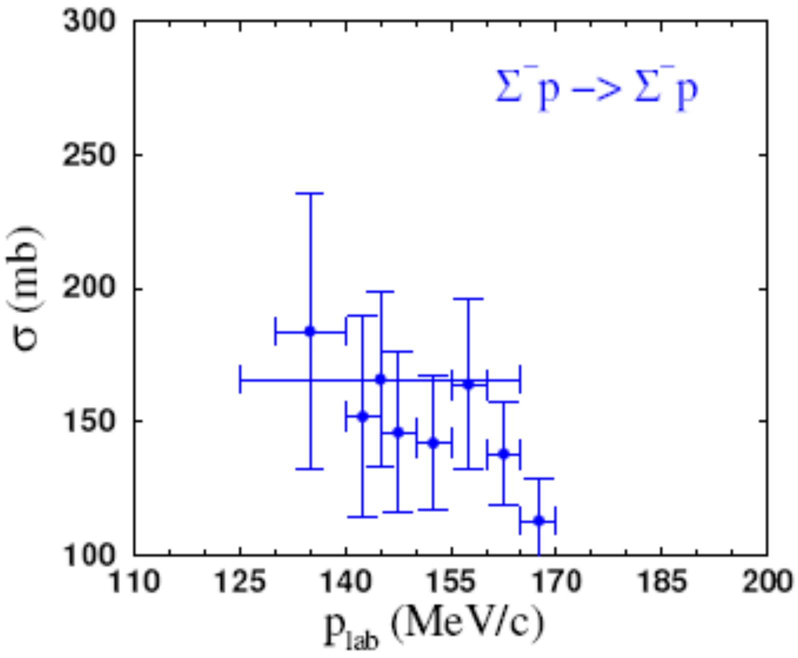

nΛ (1S0)
nΛ (3S1)
nΣ− (1S0)
nΣ− (3S1)
δ(|k| = 255 MeV) (degrees)
δ(|k| = 168 MeV) (degrees)
δ(|k| = 179 MeV) (degrees)
δ(|k| = 261 MeV) (degrees)
mπ (MeV)
EFT '06
Juelich '04
Nijmegen NSC97f

Study of the baryonic interactions in the strange sector with LQCD

provide complementary information to experiment (ΛN, ΣN, ΛΛ, ΣΣ, ΞΞ, …)

In the low energy regime, around half of the pion production theshold…





In general, YN data show large error bars and absence of true low-energy cross sections

Study of the baryonic interactions in the strange sector with LQCD

→ provide complementary information to experiment (ΛN, ΣN, ΛΛ, ΣΣ, ΞΞ, …)

In general, the analysis of data presents:
- Poor statistics
- Effective range parameters fit to data highly correlated

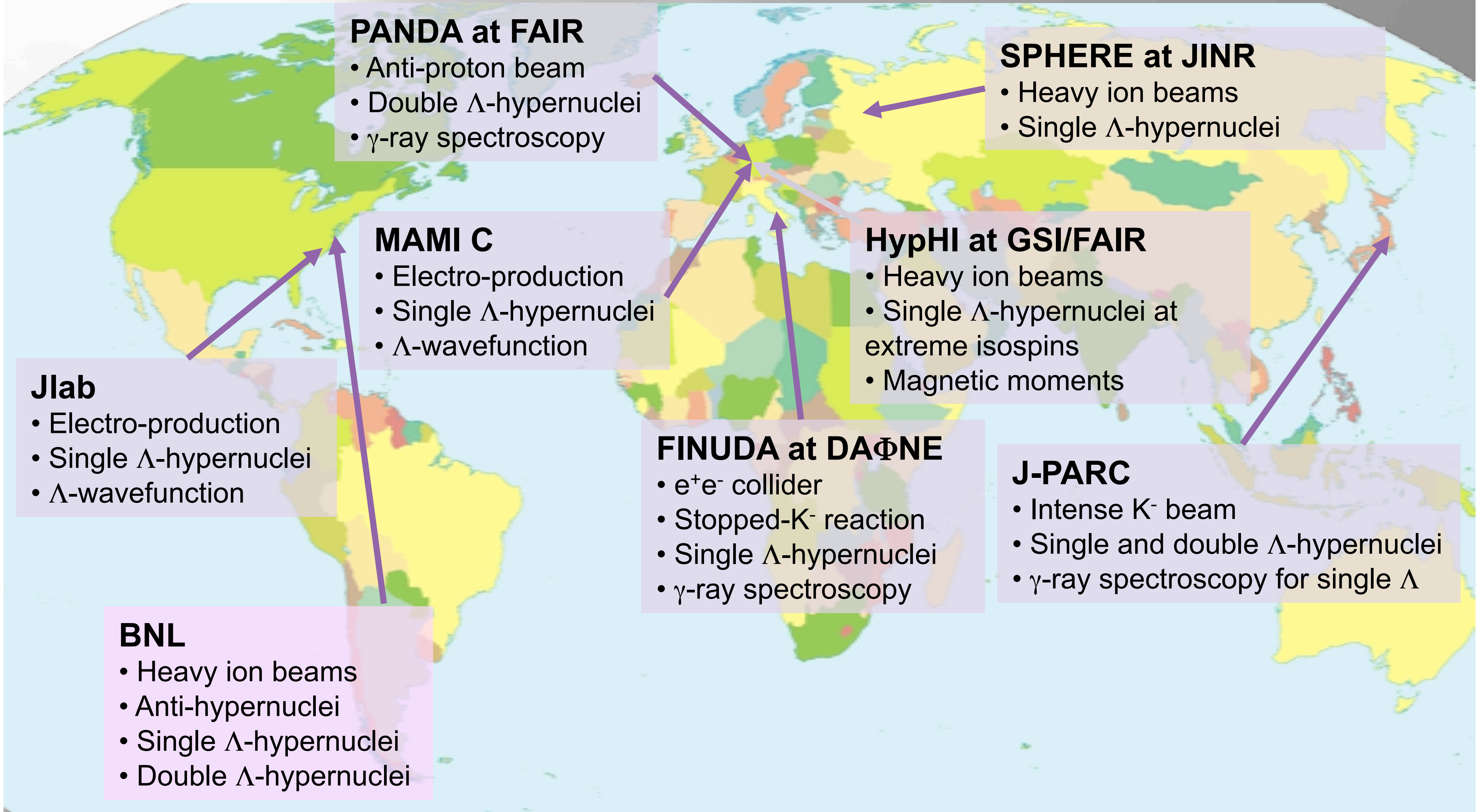
ΛN: What is safe to say?

There is not Λ–hyperdeuteron (Σ-hyperdeuteron?) → $a^{(^1S_0)} < 0, \quad a^{(^3S_1)} < 0$

Consistency of potential models with hypertriton data (b.e., spin) → $\left|a^{(^1S_0)}\right| > \left|a^{(^3S_1)}\right|$

The theoretical study of YN interactions is hindered by the lack of experimental guidance.

Experimental program



J. Pochodzalla, Int. Journal Modern Physics E, Vol 16, no. 3 (2007) 925-936

alternatives...

$p\,p \rightarrow K^+ \Lambda\, p$
(COSY, Jülich)

Balewski et al. EPJA 2 (1998)
Hinterberger, Sibirtsev, EPJA 21 (2004)
Gasparyan, Haidenbauer, Hanhart, Speth, PRC69 (2004)
Gasparyan, Haidenbauer, Hanhart, PRC72 (2005)

$\gamma\, d \rightarrow K^+ \Lambda\, n$

Gasparyan, Haidenbauer, Hanhart, K. Miyagawa

(CEBAF, ELSA, JLAB, MAMI-C)

Reconstruct the elastic two-body amplitude via the invariant mass dependence of the production amplitude in the region where the YN momentum is small.

$$K^- d \rightarrow n\Lambda\gamma$$

Gall et al., PRC42 (1990)

Gibson et al. BNL report No. 18335(1973)
Gibbs, Coon, Han, Gibson ,PRC61 (2000)

$$a(^1S_0) = -0.15 \rightarrow -5.0$$

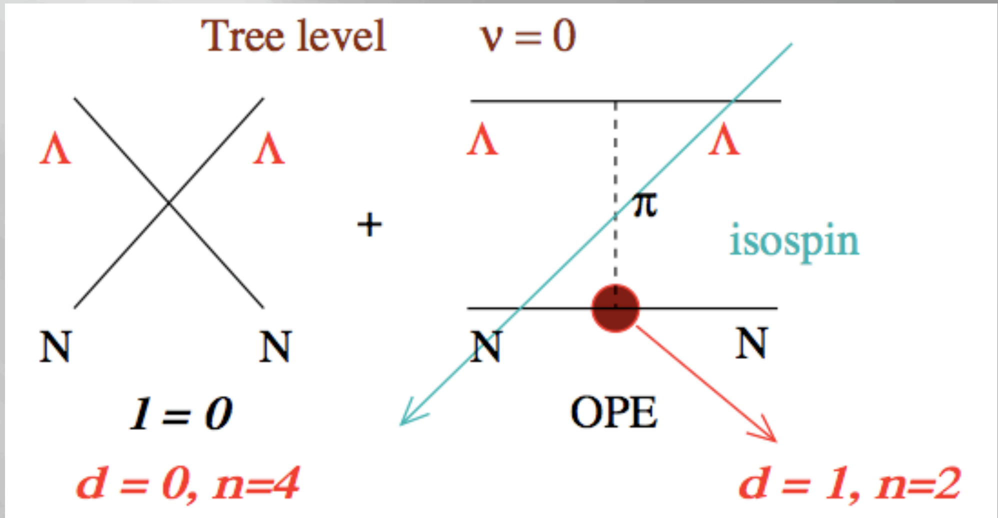
$$a(^3S_1) = -1.3 \rightarrow -2.65$$

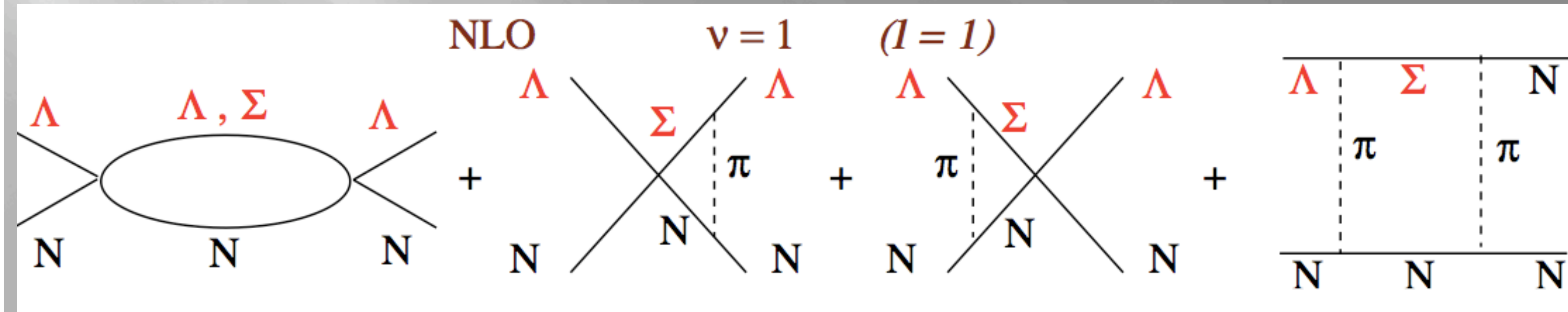
The ΛN interaction

Our (NPLQCD) first study of hyperon-nucleon interactions:

Ref: "hyperon-nucleon interactions from Lattice QCD" Nucl. Phys. A794 (2007) 62-72

Idea: write down the effective theory for the hyperon-nucleon interaction at low energies (below the pion production threshold)





Beane, Bedaque, Parreño, Savage, Nucl. Phys. A747, 55-74 (2005); nucl-th/0311027

$$a^{({}^1S_0)} = -\frac{\mu_{\Lambda N}}{2\pi}\left[{}_{\Lambda\Lambda}C_0^{({}^1S_0)} - \frac{3}{4\pi}\left({}_{\Sigma\Lambda}C_0^{({}^1S_0)}\right)^2 \mu_{\Lambda N}\,\eta + {}_{\Sigma\Lambda}C_0^{({}^1S_0)}\,\frac{3 g_{\Sigma\Lambda} g_A \mu_{\Lambda N}}{2\pi f^2}\,\frac{\eta^2 + \eta\, m_\pi + m_\pi^2}{\eta + m_\pi} - \frac{3 g_{\Sigma\Lambda}^2 g_A^2 \mu_{\Lambda N}}{4\pi f^4}\,\frac{2\eta^3 + 4\eta^2 m_\pi + 6\eta m_\pi^2 + 3 m_\pi^3}{2(\eta + m_\pi)^2} \right]$$

Extract LECs

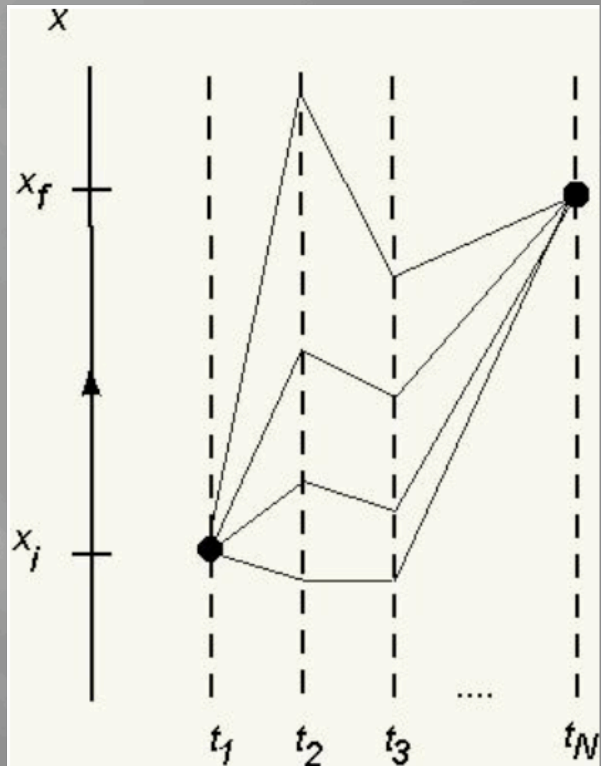
$$r^{({}^1S_0)} = -\frac{1}{\mu_{\Lambda N}\,\pi}\left[\frac{2\pi}{{}_{\Lambda\Lambda}C_0^{({}^1S_0)}}\right]^2 \left[\frac{3}{8\pi}\left({}_{\Sigma\Lambda}C_0^{({}^1S_0)}\right)^2 \frac{\mu_{\Lambda N}}{\eta} + {}_{\Sigma\Lambda}C_0^{({}^1S_0)}\,\frac{3 g_{\Sigma\Lambda} g_A \mu_{\Lambda N}}{2\pi f^2}\,\frac{3\eta^2 + 9\eta\, m_\pi + 8 m_\pi^2}{6(\eta + m_\pi)^3} - \frac{3 g_{\Sigma\Lambda}^2 g_A^2 \mu_{\Lambda N}}{4\pi f^4}\,\frac{6\eta^3 + 23\eta^2 m_\pi + 28\eta m_\pi^2 + 7 m_\pi^3}{12(\eta + m_\pi)^4} \right]$$

Result of the LQCD simulation

What is Lattice QCD ?

LQCD is a non-perturbative implementation of Field Theory, which uses the Feynman path-integral approach to evaluate transition matrix elements

The starting point is the partition function in EUCLIDEAN space-time
Imaginary time: t ➔ i τ



$$Z = \int \mathcal{D}A_\mu \, \mathcal{D}\bar{\Psi} \, \mathcal{D}\Psi \, e^{-S}$$

QCD action

$$S = d^4x \, (1/4 \, F_{\mu\nu} F^{\mu\nu} - \bar{\Psi} \, M \, \Psi)$$

By exact integration (Gaussian) on the fermion fields

$$Z = \int \mathcal{D}A_\mu \, \det M \, \exp[-d^4x \, (1/4 \, F_{\mu\nu} F^{\mu\nu} \,]$$

HARD

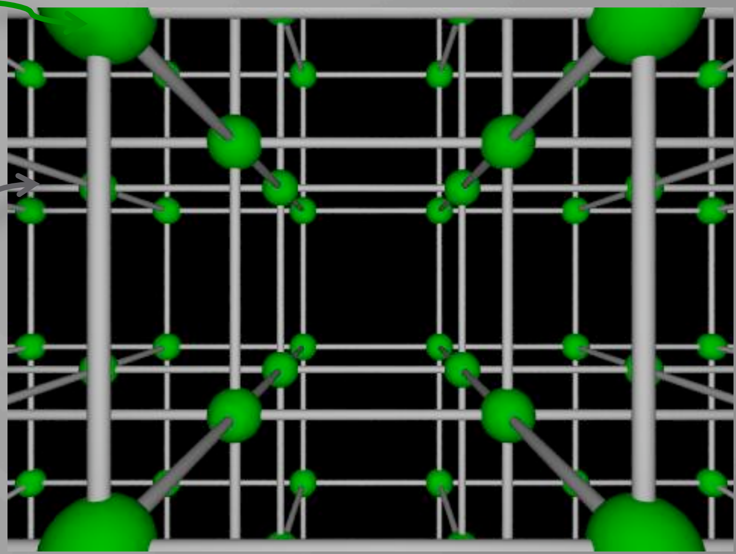
$-S_{gluon}$

nonlocal term which contains the fermionic contributions

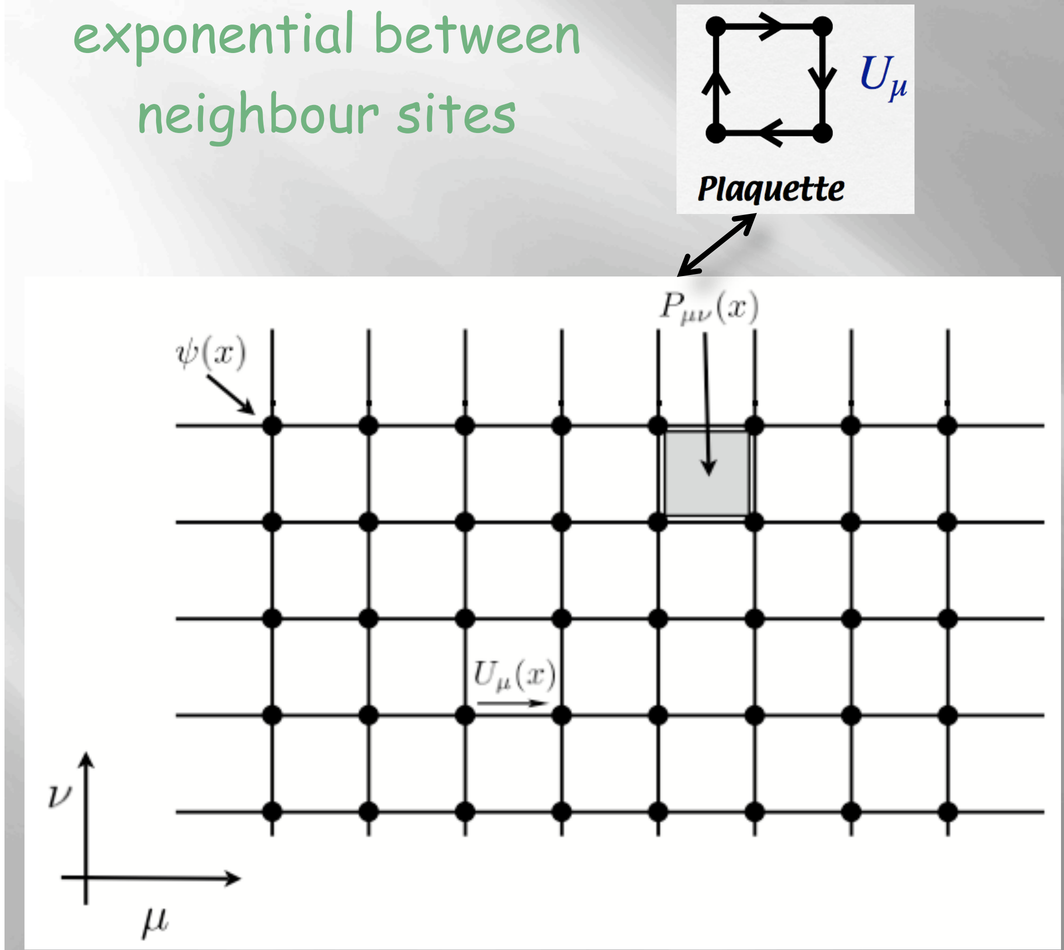
Discrete space-time

Use a discrete action

Evaluate a path ordered exponential between neighbour sites



space-time lattice



$b \to 0$; $\sum_{\{U\}}$ — continuum action

$$S_g(U) = \beta \sum_{x,\nu\mu} \left(1 - \frac{1}{3} \mathrm{Re}(Tr(P_{\nu\mu}(x)))\right)$$

$$P_{\nu\mu}(x) = U_\mu(x) U_\nu(x + \hat{\mu}) U_\mu^+(x + \hat{\nu}) U_\nu^+(x)$$

The starting point is he partition function in EUCLIDEAN space-time

$$Z = \int \prod_{\mu,x} dU_\mu(x) \prod_x d\overline{\psi} d\psi \, e^{-S_g(U) - S_f(\overline{\psi},\psi,U)}$$

$$= \int \prod_{\mu,x} dU_\mu(x) \det(D(U)^+ D(U)) \, e^{-S_g(U)}$$

$$S_f = \overline{\psi} D(U) \psi$$

Correlation functions:

$$\langle O \rangle = \frac{1}{Z} \int \prod_{\mu,x} dU_\mu(x) \, O(\frac{1}{D(U)}, U) \det(D(U)^+ D(U)) \, e^{-S_g(U)}$$

(main numerical task)
(huge integration: 8x4x6x12x6x12 x # space points)

Montecarlo Integration

Euclidean action

for real and positive actions
e^{-S}
weighting factor

$$\frac{1}{Z} \det(D(U)^+ D(U)) \, e^{-S_g(U)} \rightarrow P(U)$$ ≈ Probability

(positive definite quantity)

important sampling

Basic algorithm:

1. Produce N gauge field configurations {U} with probability distribution P(U)
2. Evaluate:

$$\langle O \rangle = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O\left(U_i, \frac{1}{D(U_i)}\right)$$

Solve a linear system of equations: $D^{+}(U)[m]\, D(U)[m]\, \chi = \phi$

Condition number ≈ 1/m

Present

Aproaching nature

$L \approx 2.5$ fm
$b \approx 0.1$ fm
$m_q \approx m_s/2$

EFT

$L \longrightarrow \infty$
$b \longrightarrow 0$
$m_q \longrightarrow m_{u,d}^{phys}$

Procedure

Configurations (MILC) ⟹ Compute propagators ⟹ Compute correlators

Sets of configurations used in our MIXED simulations

Dimensions $L_S^3 \times L_T$ (L_5 = 16)	b (fm)	L (fm)	m_π (MeV)	m_K (MeV)	no. conf x no. src
$20^3 \times 32$ m_l=0.030 m_s=0.050	0.125	2.5	591	675	564 x 24
$20^3 \times 32$ m_l=0.020 m_s=0.050	0.125	2.5	491	640	486 x 24
$20^3 \times 32$ m_l=0.010 m_s=0.050	0.125	2.5	352	595	769 x 24
$20^3 \times 32$ m_l=0.007 m_s=0.050	0.125	2.5	291	580	1039 x 24

Dimensions $L_S^3 \times L_T$ (L_5 = 12)	b (fm)	L (fm)	m_π (MeV)	m_K (MeV)	no. conf x no. src
$28^3 \times 96$ m_l=0.0062 m_s=0.031	0.09	2.5	320	560	1001 x 7
$28^3 \times 96$ m_l=0.0124 m_s=0.031	0.09	2.5	446	578	513 x 3
$40^3 \times 96$ m_l=0.0062 m_s=0.031 (L_5 = 40)	0.09	2.5	230	539	109 x 1
$40^3 \times 96$ m_l=0.0062 m_s=0.031 (L_5 = 12)	0.09	2.5	234	540	109 x 1

2+1 flavors

Domain-Wall valence quarks on staggered sea quark configurations

One hadron in a box
Extracting masses

Lattice simulations → Evaluation of vacuum correlation functions:

$$\langle \Gamma_1(t)\Gamma_2(0)\rangle \equiv \langle 0|\Gamma_1(t)\Gamma_2(0)|0\rangle \qquad \text{at large t}$$

$$\langle \Gamma_1(t)\Gamma_2(0)\rangle = \langle 0|\Gamma_1(0)\, e^{-\hat{H}t}\, \Gamma_2(0)|0\rangle = \sum_n \langle 0|\Gamma_1(0)|E_n\rangle e^{-E_n t} \langle E_n|\Gamma_2(0)|0\rangle$$

$$\rightarrow \quad \langle 0|\Gamma_1(0)|E_0\rangle\langle E_0|\Gamma_2(0)|0\rangle e^{-E_0 t}, \qquad \text{as} \quad t \to \infty$$

lowest energy eigenstate

from the exponential decay → energies

Ensure that the (asymptotic) exponential dominates the correlation function

Ex:

$$C_{\pi^+}(t) = \sum_{\vec{x}} \langle \pi^-(t,\vec{x})\pi^+(0,\vec{0})\rangle, \qquad \pi^+(t,\vec{x}) = \bar{u}(t,\vec{x})\gamma_5 d(t,\vec{x})$$

Extracting masses and energy shifts

$$p_i(t,\vec{x}) = \varepsilon_{abc}\, d_i^a(t,\vec{x})\,(\, d^{bT}(t,\vec{x})\, C\,\gamma_5\, u^c(t,\vec{x})\,)$$

One-baryon correlator: **mass**

$$C_A(t) = \sum_{\vec{x}} \left\langle A(t,\vec{x})\, A^\dagger(0,\vec{0}) \right\rangle = \sum_n C_A^n\, e^{-E_A^n t} \rightarrow C_A\, e^{-M_A t}$$

2-baryon correlator:

$$C_{AB}(t) = \sum_{\vec{x},\vec{y}} \left\langle A(t,\vec{x})\, B(t,\vec{x})\, B^\dagger(0,\vec{0})\, A^\dagger(0,\vec{0}) \right\rangle = \sum_n C_{AB}^n\, e^{-E_{AB}^n t} \rightarrow C_{AB}\, e^{-E_{AB} t}$$

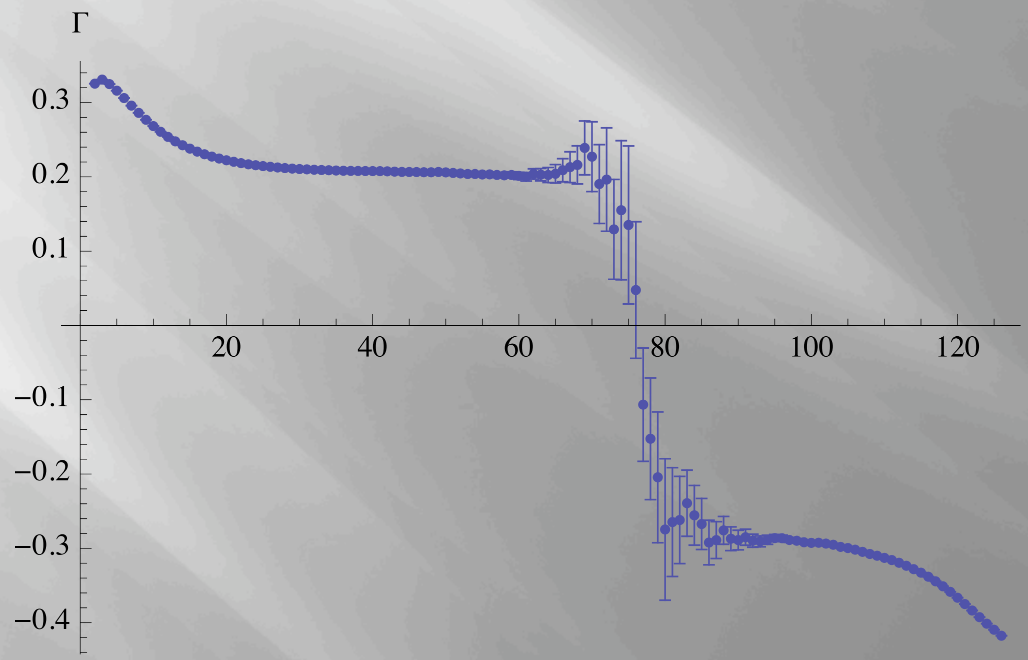
Energy shift: $\Delta E = E_{AB} - M_A - M_B$

$$G_{AB}(t) = \frac{C_{AB}(t)}{C_A(t)\, C_B(t)} = \sum_n C^n e^{-\Delta E^n t} \rightarrow C e^{-\Delta E\, t}$$

One hadron in a box

generalized effective mass plots $$M_{eff,t_J} = \frac{1}{t_J}\log\left(\frac{C(t)}{C(t+t_J)}\right) \to M_0$$

(statistical average over measurements on an ensemble of configurations)



proton

clover on clover, 20^3x128, antiperiodic BC in t direction
smeared-point, 1194 conf

$$\Delta E \equiv \sqrt{p^2 + M_A^2} + \sqrt{p^2 + M_B^2} - M_A - M_B$$

(below inelastic thresholds)

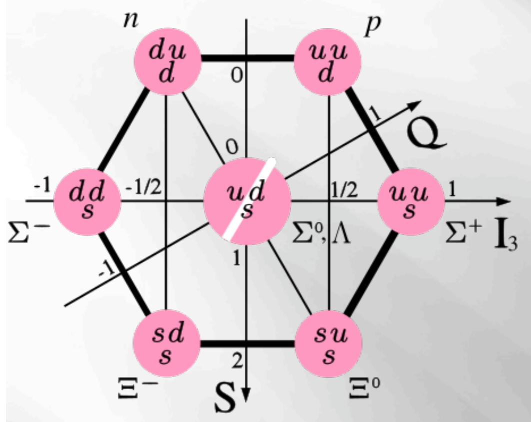
obtained from the simulation

$$S\left(\eta^2 = \frac{p^2 L^2}{4\pi^2}\right) \equiv \sum_{j}^{|\vec{j}|<\Lambda} \frac{1}{|\vec{j}|^2 - \eta^2} - 4\pi\Lambda$$

u.v. regulator

$$-\frac{1}{a} + \frac{1}{2} r_0 p^2 = \quad p \cot\delta(p) = -\frac{1}{\pi L} S\left(\frac{p^2 L^2}{4\pi^2}\right)$$

studied BB channels in the strange sector



channel	isospin	isospin projection	quark content	strangeness
Λn	1/2	-1/2	uuddds	-1
Σ^-n	3/2	-3/2	uddddds	-1
$\Lambda\Lambda$	0	0	uuddss	-2
$\Sigma^+\Sigma^+$	2	2	uuuuss	-2
$\Xi^0\Xi^0$	1	1	uussss	-4

not considered in the present work

channel	isospin	isospin projection	quark content	strangeness	mixing
Ξ^0n	0	0	uuddss	-2	$\Lambda\Lambda$
Ξ^0n	1	0	uuddss	-2	$\Sigma^0\Lambda$
Ξ^0p	1	1	uuudss	-2	$\Sigma^+\Lambda$
Ξ^-n	1	-1	udddss	-2	$\Sigma^-\Lambda$

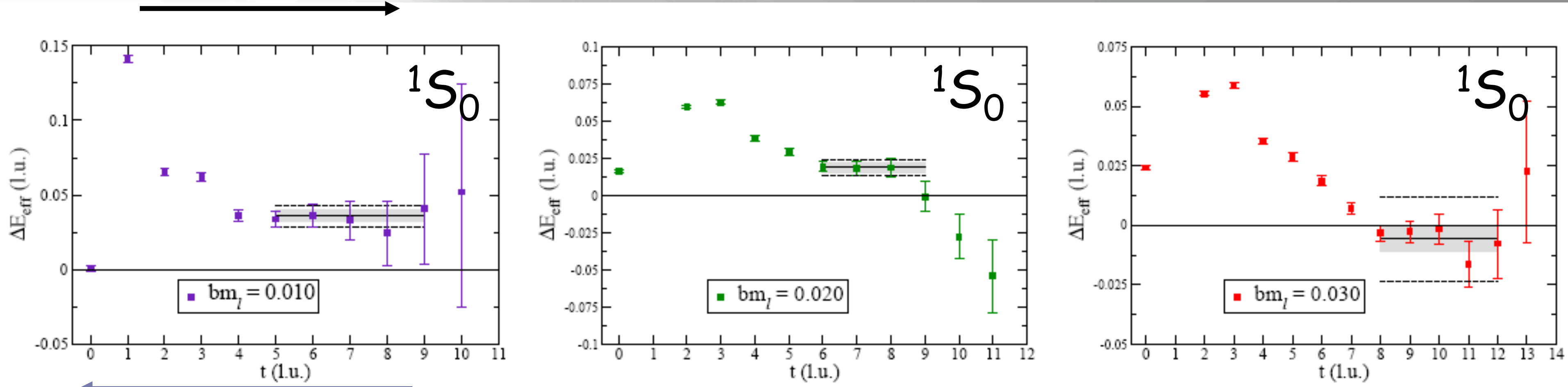
Λn

NPLQCD, Nucl. Phys. A794 (2007) 62-72

MILC $20^3 \times 32$ L = 2.5 fm b ~ 0.125 fm

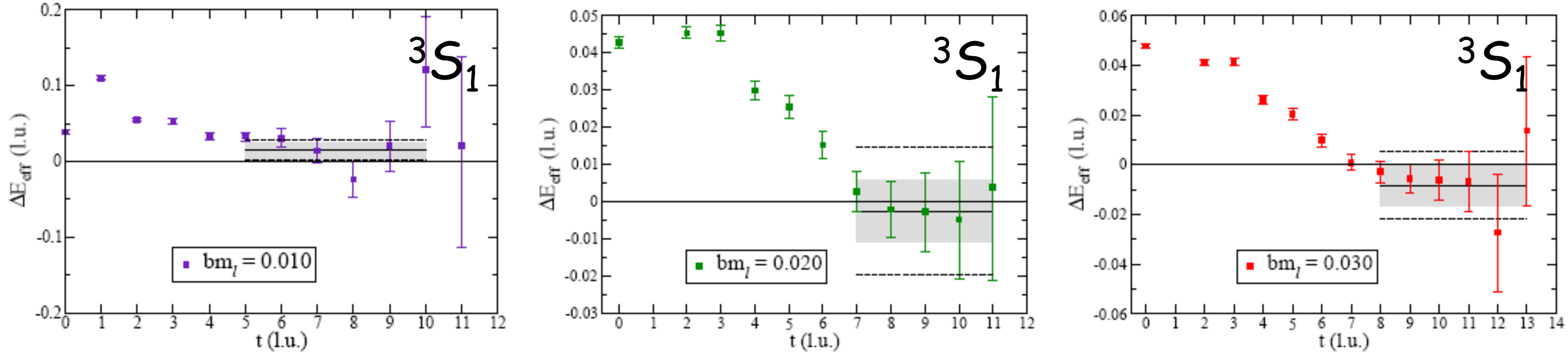
signal-to-noise ratio ~ $\sqrt{N_{conf}}\, e^{-(M_N + M_\Lambda - 2m_\pi - m_K)t}$

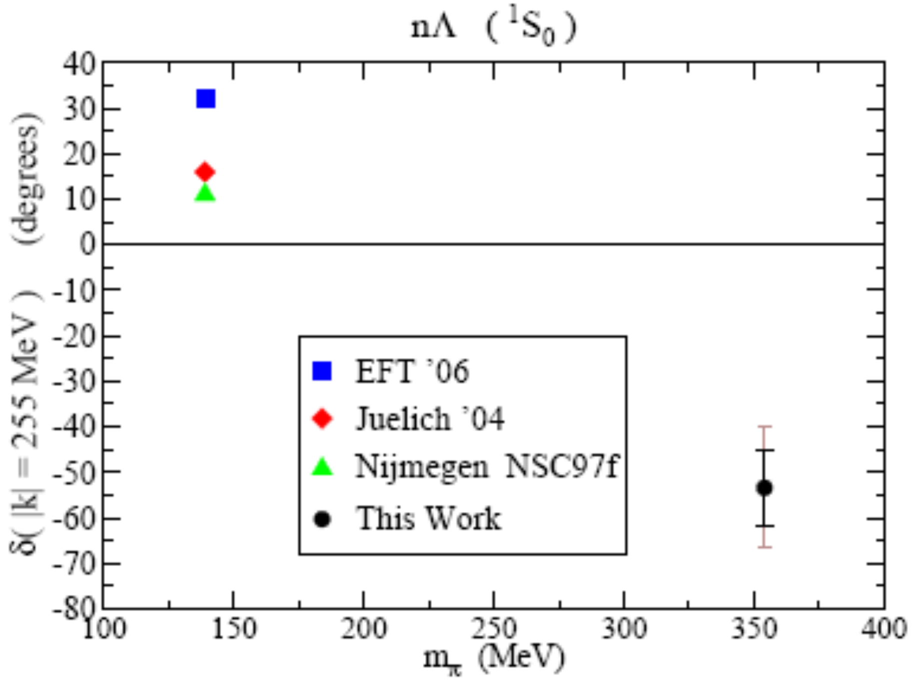
$$\frac{G_{AB}(t)}{G_{AB}(t+1)} = \Delta E$$

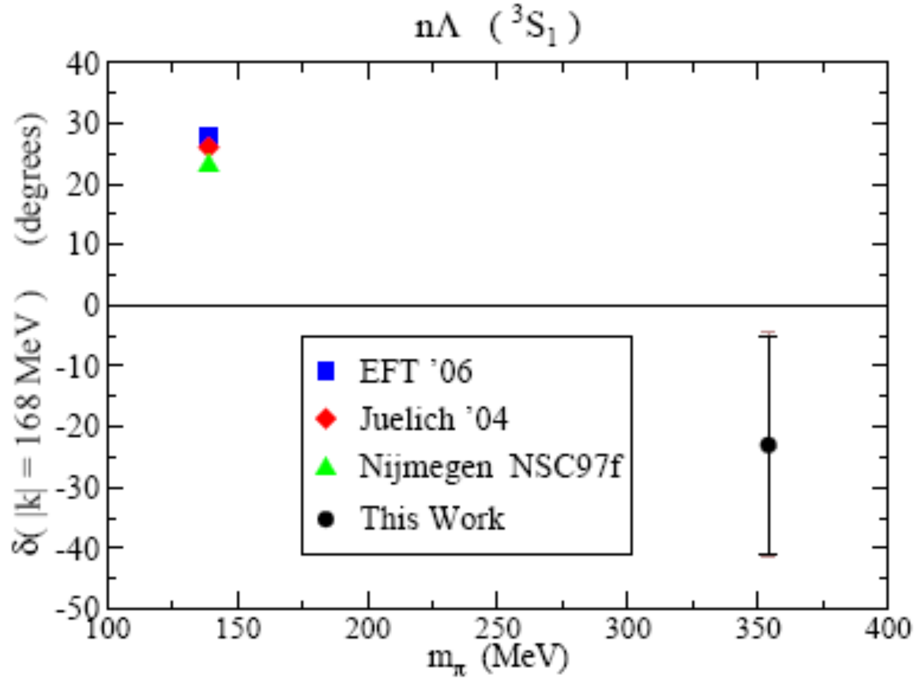


contamination from excited states

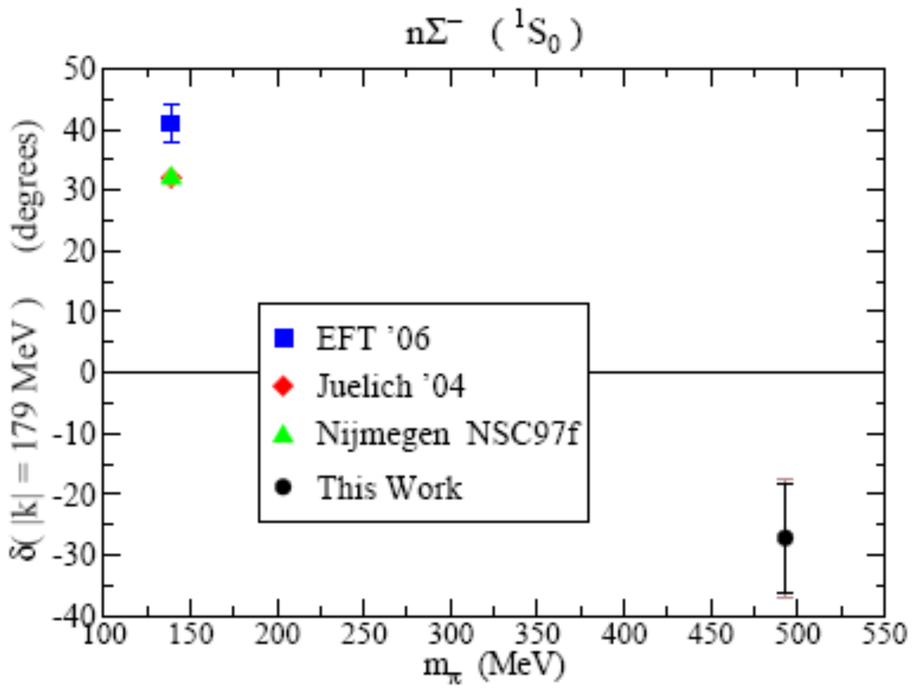
m_π = 350 MeV m_π = 490 MeV m_π = 590 MeV

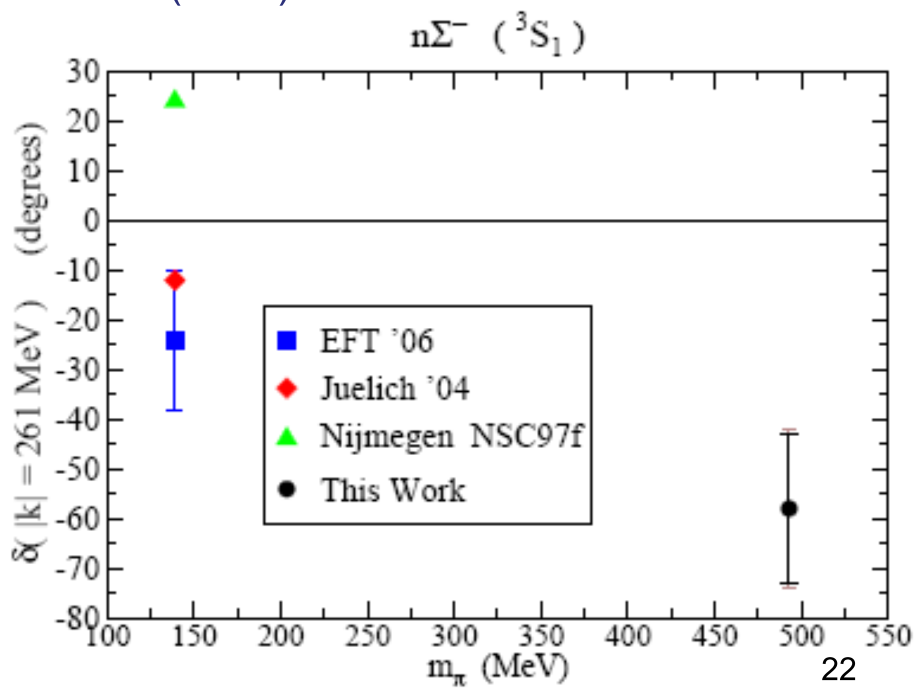






NPLQCD, Nucl. Phys. A 794 (2007) 62-72





more recently: no MIXING: high statistics simulations

Anisotropic ($b_s > b_t$) clover lattices
⟹ higher resolution in the time direction
i.e. better study of noisy states

- 292500 sets of measurements
- 1194 gauge configurations of size $20^3 \times 128$

produced by the Hadron Spectrum Collaboration
- anisotropy parameter $\xi = b_s/b_t = 3.5$
- spatial lattice spacing of $b_s = 0.1227 \pm 0.0008$ fm
- $M_\pi \approx 390$ MeV

ADVANTAGES

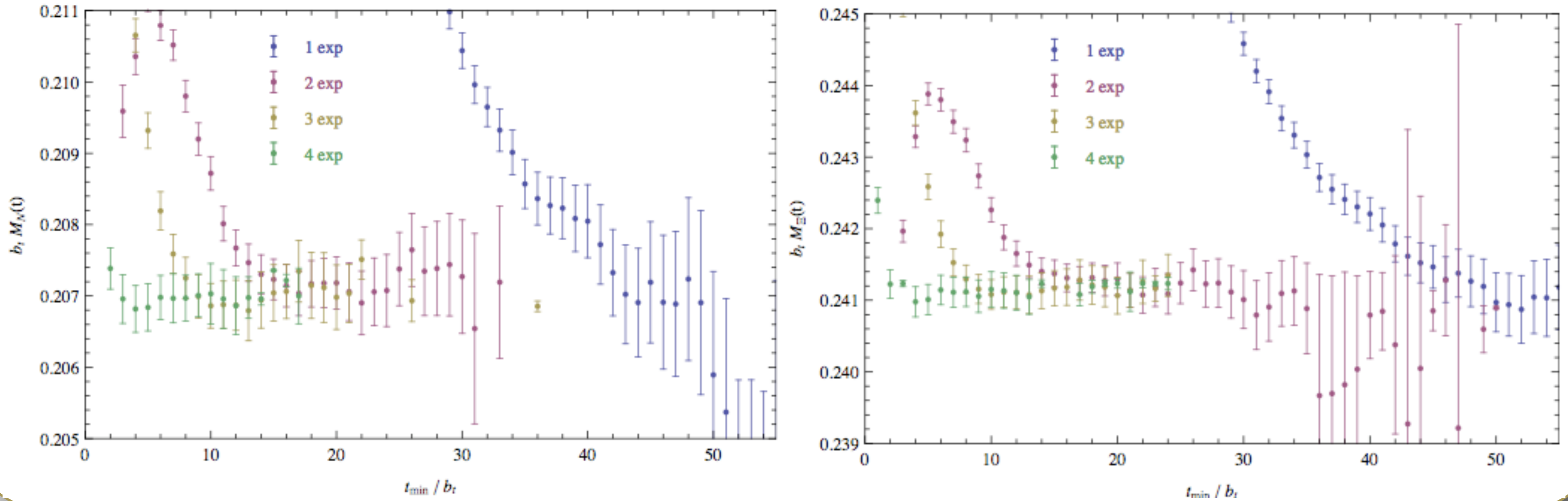
No mixed-action calculation: we used the same fermion action used in the gauge-field generation to compute the quark propagators ⟶ clover on clover

Faster than our previous MA simulations DW on staggered (4-D clover compared to 5-D DW fermions)

Clover discretization keeps corrections *O(b)*

Clover discretization does not have a lattice chiral symmetry… systematic uncertainties in the properties/interaction of baryons?

One hadron simulations



M_π = 390.3(0.7)(0.3) MeV
M_N = 1163.9(1.8)(0.6) MeV
M_Σ = 1283.7(1.6)(1.0) MeV

M_K = 546.0(0.6)(0.2) MeV
M_Λ = 1252.4(1.6)(0.3) MeV
M_Ξ = 1356.1(1.4)(0.2) MeV

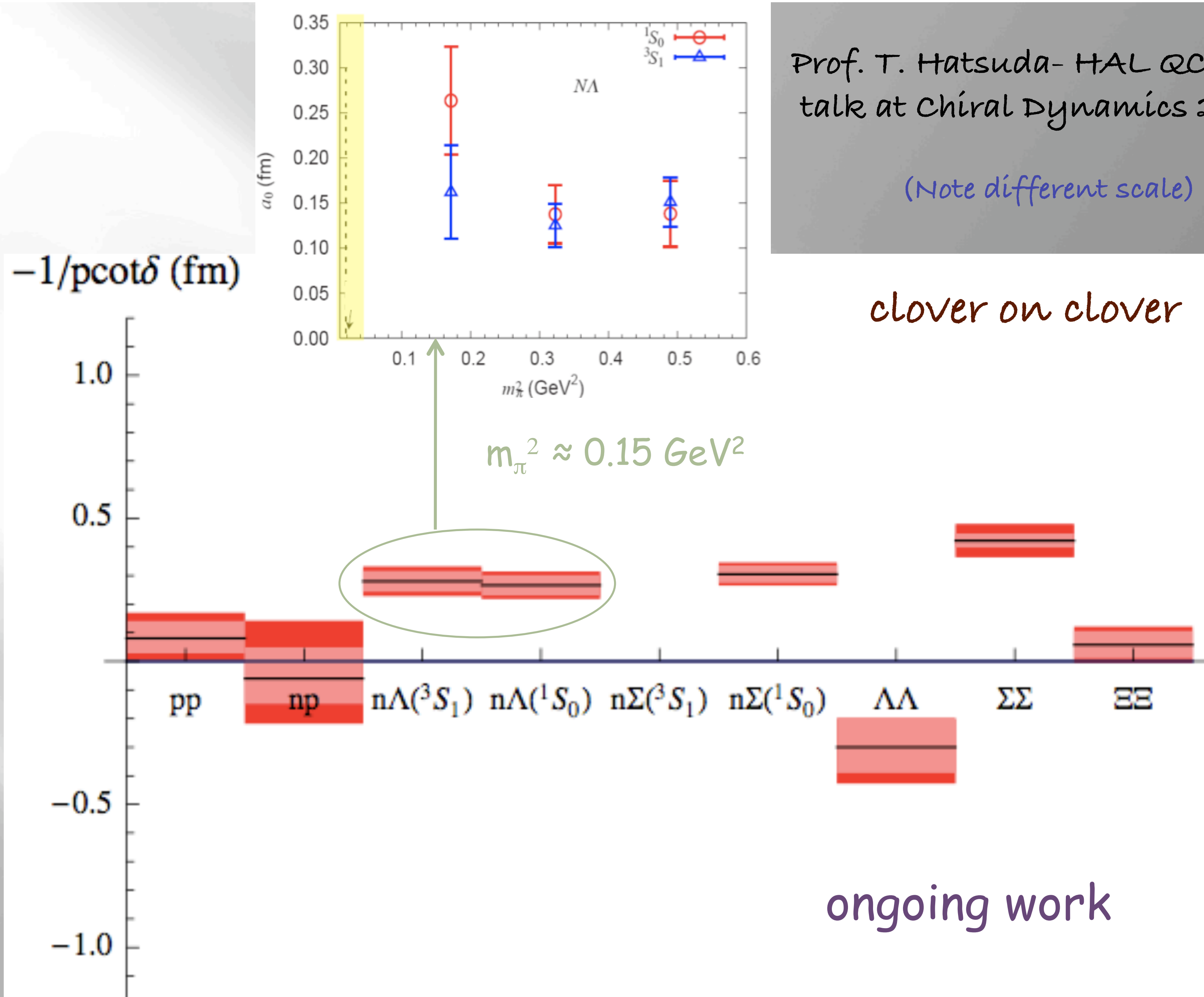
$E_{N(1/2^-)}$ = 1610(06)(11) MeV
$E_{\Sigma(1/2^-)}$ = 1727(06)(06) MeV

$E_{\Lambda(1/2^-)}$ = 1679(05)(02) MeV
$E_{\Xi(1/2^-)}$ = 1825(6)(5) MeV

NPLQCD, Phys. Rev. D79 (2009) 114502

Prof. T. Hatsuda- HAL QCD Coll talk at Chiral Dynamics 2009 (Bern)

(Note different scale)



Preliminary

meson-baryon

Particles	Isospin	Quark Content
$\pi^+\Sigma^+$	2	$uud\bar{s}$
$\pi^+\Xi^0$	3/2	$ud\bar{d}ss$
K^+p	1	$uud\bar{s}$
K^+n	0 *and* 1	$udd\bar{s}$
$\overline{K}^0\Sigma^+$	3/2	$ud\bar{d}ss$
$K^0\Xi^0$	1	$u\bar{d}sss$

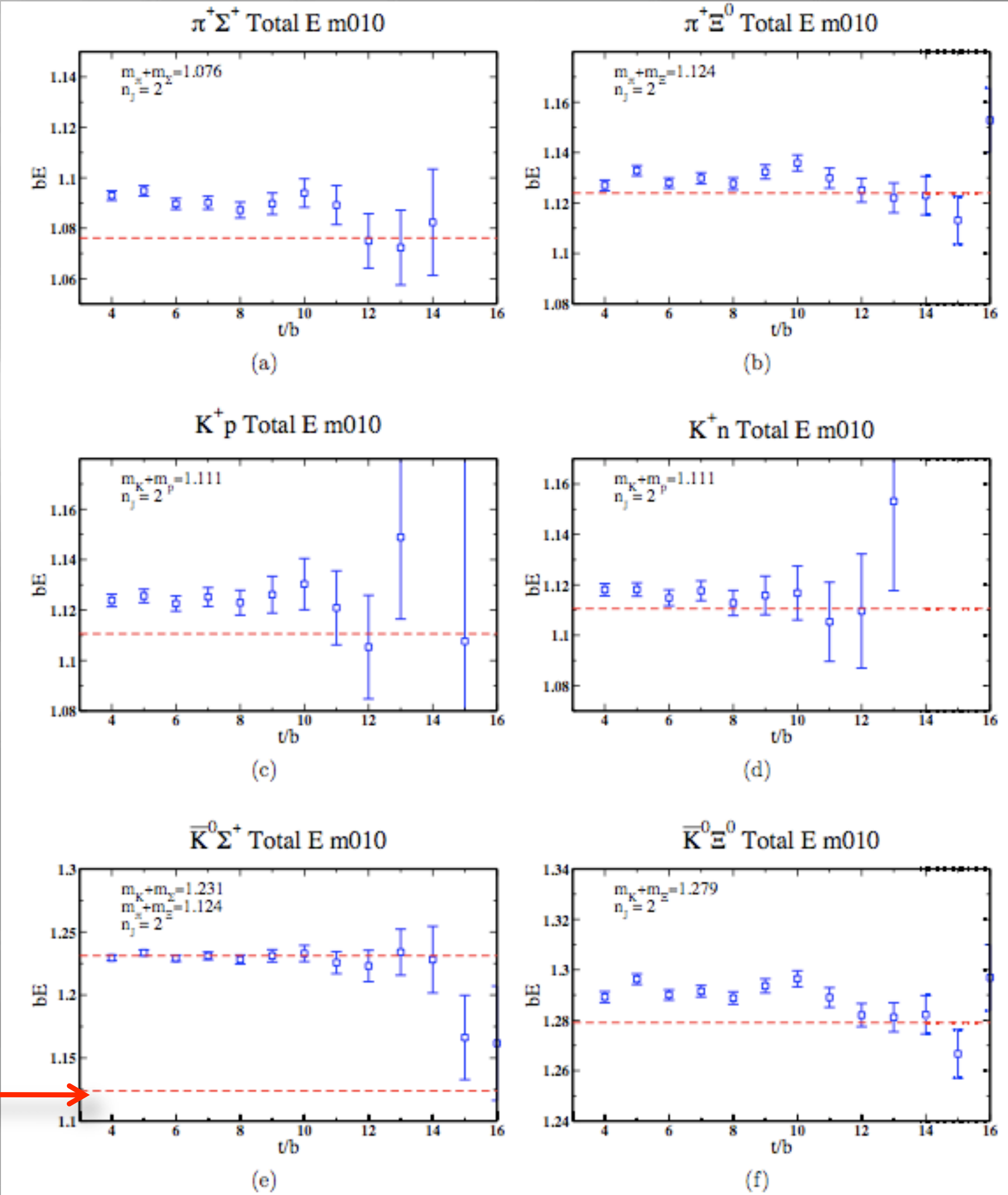
(no anihilation diagrams)

$$C_{\phi B}(t) = P_{ij} \sum_{\vec{x},\vec{y}} \left\langle \phi^+(t,\vec{x}) \overline{B}_i(t,\vec{y}) \phi(0,\vec{0}) B_j(0,\vec{0}) \right\rangle$$

$$E^{eff}_{\phi,B} = \frac{1}{n_J} \log\left(\frac{C_{\phi,B}(t)}{C_{\phi,B}(t+n_J)} \right)$$

$C^{(SS)} - \alpha\, C^{(SP)}$

$\pi^+\ \Xi^0$ →



NPLQCD, arXiv:0803.2728v1 [hep-lat]

Three Baryons

Wick contractions to form the correlation function is naively $N_u!\,N_d!\,N_s!$

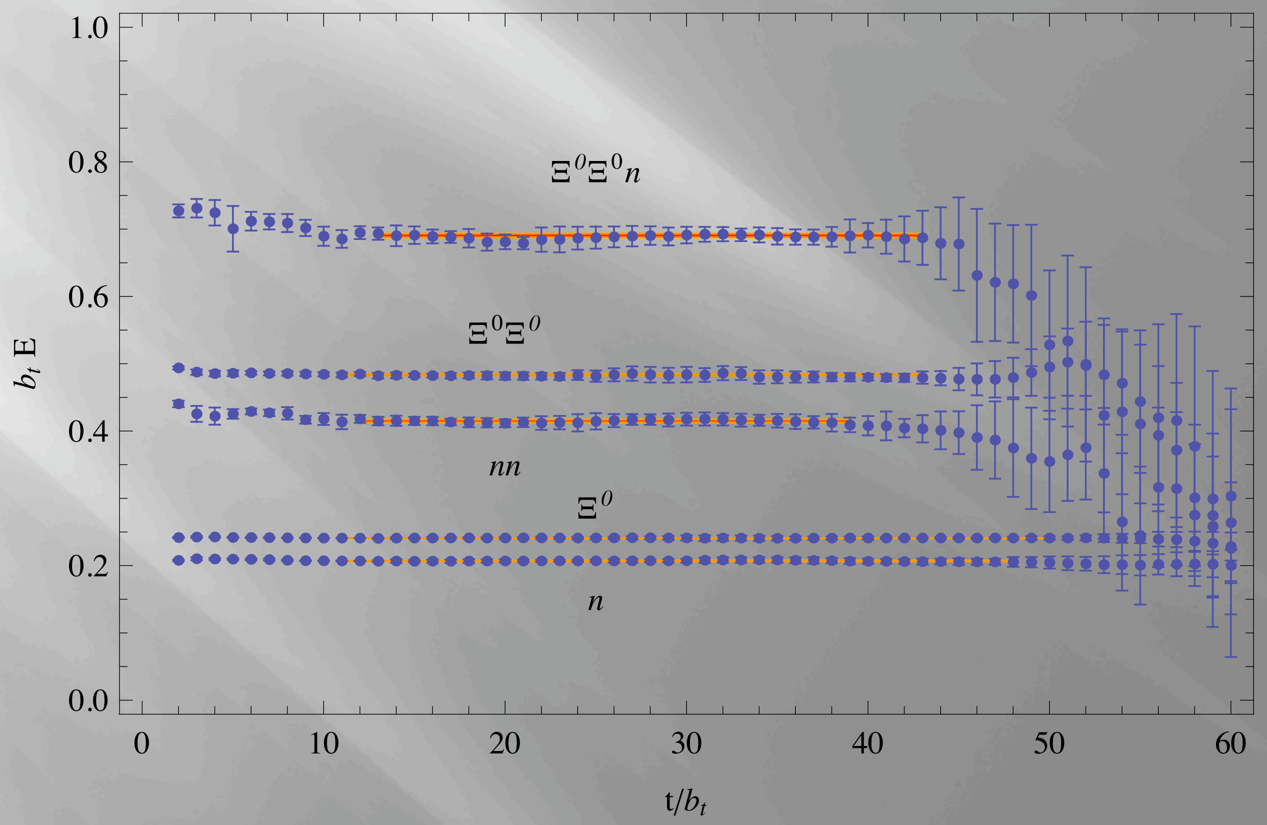
→ the cheapest 3-baryon system would be $\Xi^0\,\Xi^0\,n$, with 3! 2! 4! = 288 Wick contractions

The $\Lambda\Lambda\Sigma^0$ requires 6^3 contractions but the signal is less clear due to the difference in N_s

(Note that the triton, with N_u=4 and N_d=5 requires 2880)

Three Baryons

energy splitting $\overline{G}_{\Xi^0\Xi^0 n}(t) = \frac{\overline{C}_{\Xi^0\Xi^0 n}(t)}{\overline{C}^2_{\Xi^0}(t)\overline{C}_n(t)} \longrightarrow A_0 e^{-\delta E_{\Xi^0\Xi^0 n} t}$

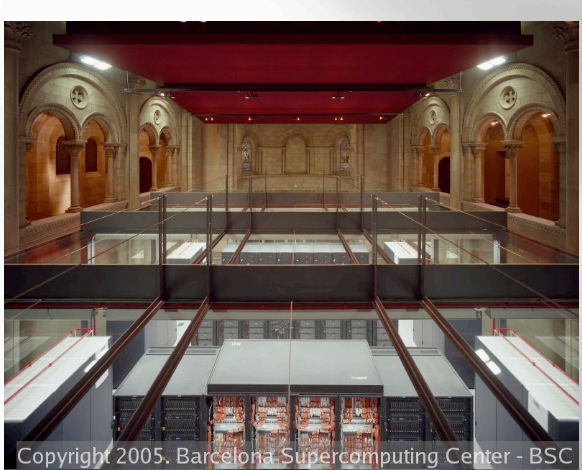


I did not cover…

1. How does the noise-to-signal scale in hadron correlators?

2. How to distinguish between scattering states and bound states?

acknowledgents…computational resources



BSC Barcelona Supercomputing Center
Centro Nacional de Supercomputación

Fermilab

Jlab

USQCD US Lattice Quantum Chromodynamics

NERSC

Franklin - Cray XT4
LBNL

NCSA

U Illinois

TeraGrid

NSF-LLNL

INT
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in memory of Prof. Cornelius Bennhold

Over the years, Cornelius' thorough vision of the field, together with his open minded attitude and generosity in offering advise, has guided scientists through unexplored and imaginative research paths, leading to the present impressive knowledge and understanding of the mechanisms governing the decay of hypernuclei.